Experiment 4
Angle Modulation-Part II

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FM MODULATION

- Angle Modulation: is a modulation technique where the amplitude of the carrier signal is held constant while either the phase or the time derivative of the phase is varied linearly with the message signal m(t).
- An FM signal is expressed as:

$$s(t) = A_c \cos\left(\omega_c t + 2\pi k_f \int_{-\infty}^{t} m(\alpha)d\alpha\right)$$

- k_f : sensitivity of the FM modulator in Hz/V
- A_c: The amplitude of the carrier.

FM MODULATION

- The instantaneous frequency of s(t) is:

$$f_i(t) = f_c + k_f m(t)$$

- Note that this frequency is linearly proportional to the message signal m(t).
- The FM modulation index is defined as the peak frequency deviation divided by the message bandwidth.

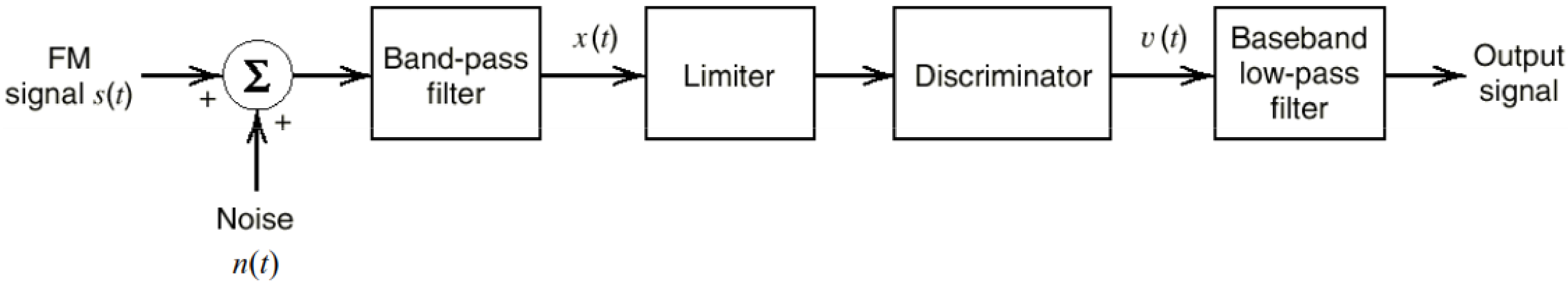
$$\beta = \frac{\Delta f}{f_m}$$

- When $m(t) = A_m \cos \omega_m t$

$$s(t) = A_c \cos\ (\omega_c t + \beta \sin 2\pi f_m t).$$

$$f_i(t) = f_c + A_m k_f \cos 2\pi f_m t$$

FM Receiver



- Bandpass filter: removes any signal outside the bandwidth of $f_c \pm B_T / 2$
  - the predetection noise at the receiver is filtered with a filter with BW B_T
- FM signal has a constant envelope , hence use a limiter to remove any amplitude variations
- Discriminator: a device with output proportional to the deviation in the instantaneous frequency. It recovers the message signal.
- Final baseband low pass filter has a bandwidth of *W . It passes the message signal and removes out of band noise.*

The discriminator

- The input-output relationship of a discriminator is:

$$y(t) = \frac{1}{2\pi} k_D \frac{d\theta}{dt}$$

- When $s(t) = A_c \cos\big(\omega_c t + \theta(t)\big)$, then

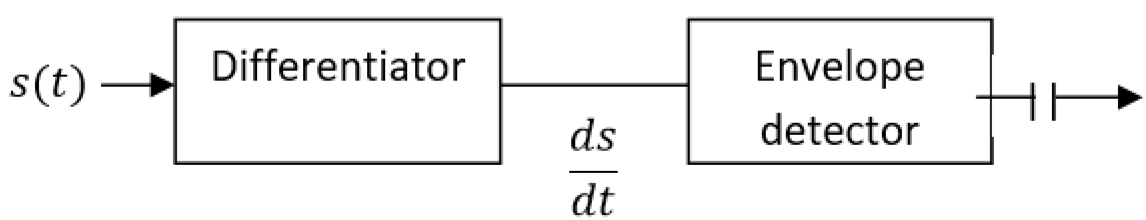
$$y(t) = k_D k_f \, m(t)$$

- One practical realization of a discriminator is a differentiator followed by an envelope detector.

$$\frac{ds(t)}{dt} = -A_c\left(\omega_c + \frac{d\theta}{dt}\right)\sin\big(\omega_c t + \theta(t)\big)$$

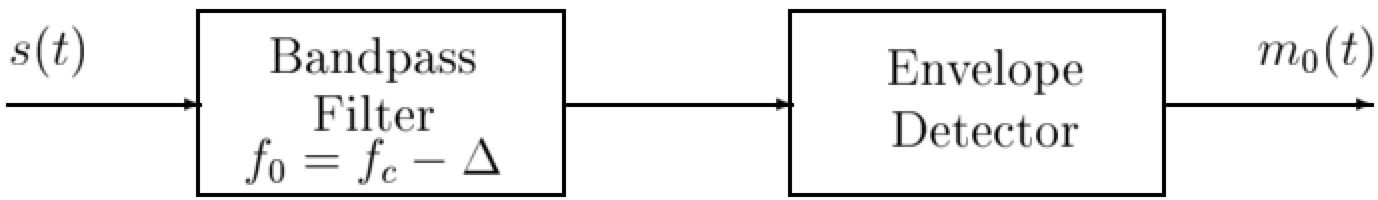
- The output of the envelope detector is $A_c\left|\left(\omega_c + \frac{d\theta}{dt}\right)\right|$
- The capacitor blocks the DC term and so the output is:

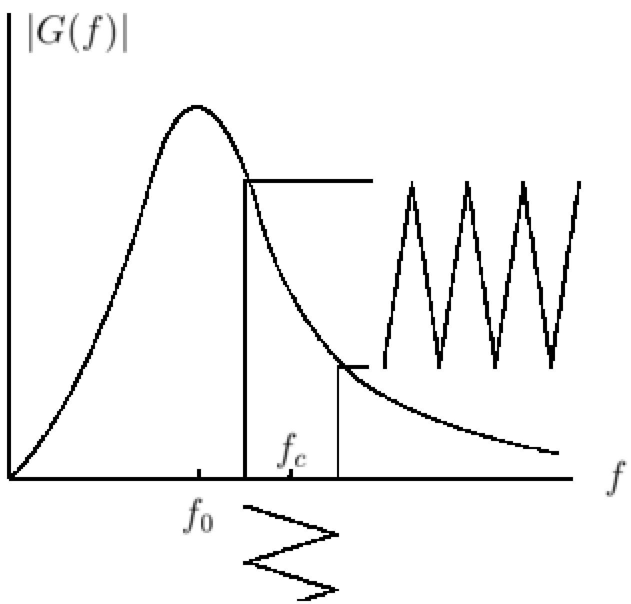
$$V_0 = A_c \frac{d\theta}{dt} = 2\pi k_f \, A_c m(t)$$



FM DEMODULATION

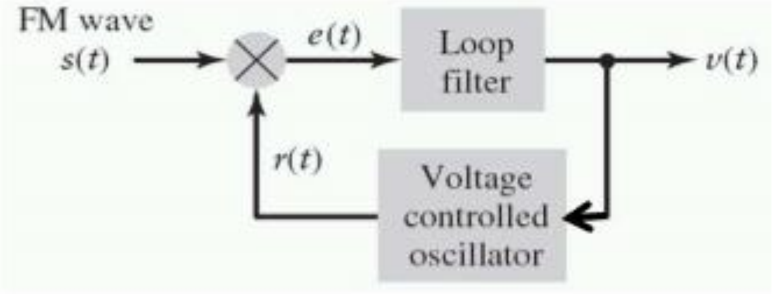
- Differentiation can be performed using a bandpass filter where the transfer function behaves as a linear function in f G(f) = -jkf.





The Phase Locked Loop

- PLL: a negative feedback system that locks on a signal and changes with it slightly as it changes.
- Has many applications in communications:
  - Carrier synchronization: Costa receiver
  - Demodulation: e.g., DSB, FM
  - Frequency multiplication and division



- Three main components:
  - Phase detector (Multiplier)
  - Loop filter: low pass filter
  - Voltage controlled oscillator (VCO): a FM system
- The VCO is designed such that when the input v(t) of VCO is 0, the following conditions are satisfied:
  - The frequency of the VCO is the unmodulated carrier frequency (also called free running frequency)
  - The VCO output has a 90 degree phase shift from the unmodulated carrier

The Phase Locked Loop

The input FM signal is:

- $s(t) = A_c \sin(2\pi f_c t + \phi_1(t))$ $\phi_1(t) = 2\pi k_f \int_0^t m(\tau) d\tau$

The VCO output is:

The VCO performs freq modulation on its own control signal v(t), so the phase is:

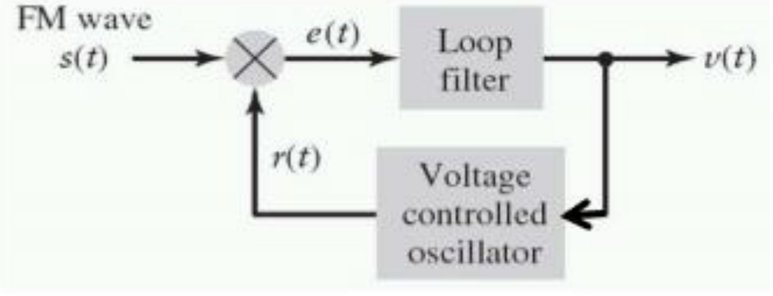
$$s(t) = A_c \sin(2\pi f_c t + \phi_1(t)) \qquad r(t) = A_v \cos(2\pi f_c t + \phi_2(t))$$

$$\phi_1(t) = 2\pi k_f \int_0^t m(\tau) d\tau \qquad \phi_2(t) = 2\pi k_v \int_0^t v(\tau) d\tau$$

PLL Objectives: The function of the feedback loop around the VCO is to adjust the phase $\phi_2(t)$ of VCO output so that it equals $\phi_1(t)$:

If $\phi_e(t) \approx 0$ or $\phi_1(t) \approx \phi_2(t)$,

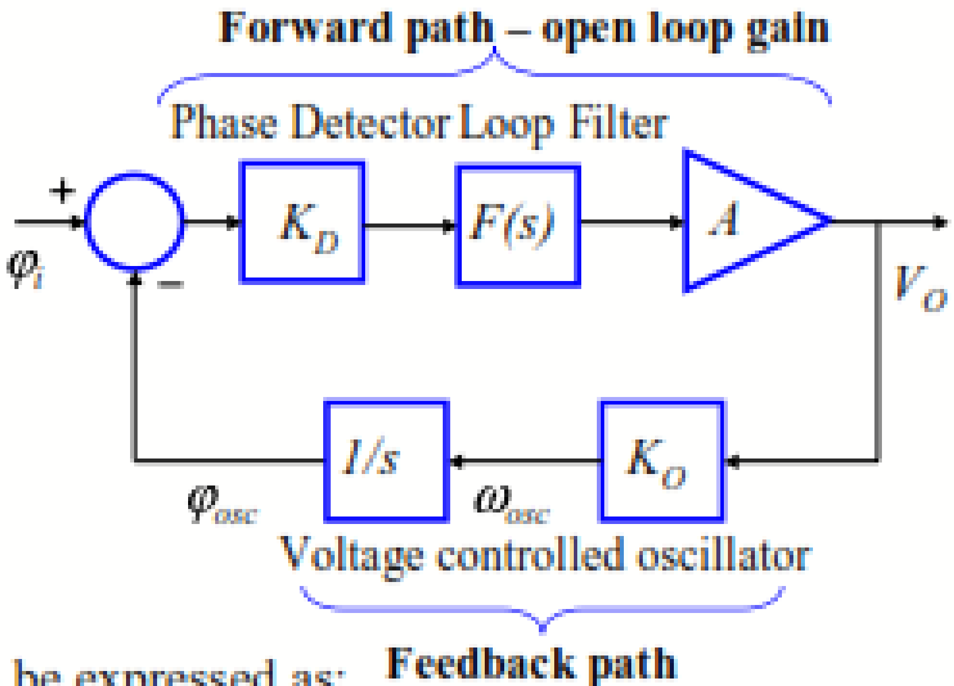
V(t) will be proportional to m(t).
Hence PLL can perform demodulation



First order PLL

The closed-loop gain transfer function is given by

$$\frac{V_O}{\varphi_i} = \frac{K_D F(s) A}{1 + K_D F(s) A \cdot (K_O / s)}$$



Assuming

$$\omega_i = \frac{d\varphi_i}{dt} \quad \text{then} \quad \omega_i = s\varphi_i(s)$$

The transfer function in terms of frequency variations therefore can be expressed as:

$$\frac{V_O}{\omega_i} = \frac{1}{s}\frac{V_O}{\varphi_i} = \frac{K_D F(s) A}{s + K_D F(s) A \cdot K_O}$$

Assuming that LP is removed and $K_v = K_O K_D A$, hence

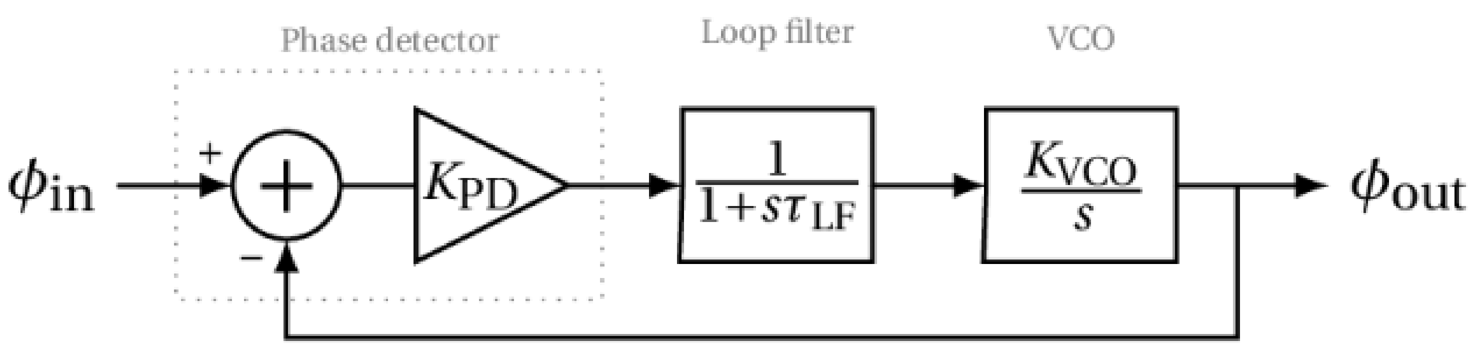
$$\frac{V_O}{\omega_i} = \frac{K_v}{s + K_v}\frac{1}{K_O}$$

Thus the loop inherently produces a first order low-pass transfer characteristic

Linear Phase Locked loop

Below, is a second order PLL. The dynamics of the loop can be controlled by varying the loop gain τ. We will consider two cases. The first keeps the loop in the linear region and the second drives the loop into the nonlinear region.

Linear analysis of the PLL in terms of phase, $H(s) = \phi_{out}(s) / \phi_{in}(s)$.



$$H(s) = \frac{K_{\mathrm{PD}} K_{\mathrm{VCO}}}{\tau_{\mathrm{LF}} s^2 + s + K_{\mathrm{PD}} K_{\mathrm{VCO}}}$$

Frequency response of the PLL

- In this experiment, the PLL will be used as a frequency and phase demodulator.
- We will investigate the frequency response of the PLL for two values of the loop filter gain τ_1 and τ_2 when the PLL is used as an FM demodulator.
- This is obtained by varying the message frequency and calculating the ratio of the PLL output to the input signal for each value of f (in the frequency domain)
- You will be required to find the bandwidth of the loop.
- Is there a major difference between the frequency responses corresponding to τ_1 and τ_2 ?

Discriminator Output

- The discriminator output in the presence of both signal and noise:

$$k_f m(t) + \frac{1}{2\pi A}\frac{dn_s(t)}{dt}$$

A component proportional to message and a component proportional to the derivative of the noise

- What is the PSD of

$$n_d(t) = \frac{dn_s(t)}{dt}$$

- Fourier theory:

$$\text{if} \quad x(t) \leftrightarrow X(f)$$

$$\text{then} \quad \frac{dx(t)}{dt} \leftrightarrow j2\pi f X(f)$$

- Differentiation with respect to time = passing the signal through a system with transfer function of $H(f) = j2\pi f$

Discriminator Output

$$S_o(f) = |H(f)|^2 S_i(f)$$

- $S_i(f)$: PSD of input signal
- $S_o(f)$: PSD of output signal
- $H(f)$: transfer function of the system

- Then: $\{\text{PSD of } n_d(t)\} = |j2\pi f|^2 \times \{\text{PSD of } n_s(t)\}$

$$\{\text{PSD of } n_s(t)\} = \left\{ N_0 \text{ within band} \pm \frac{B_T}{2} \right\}$$

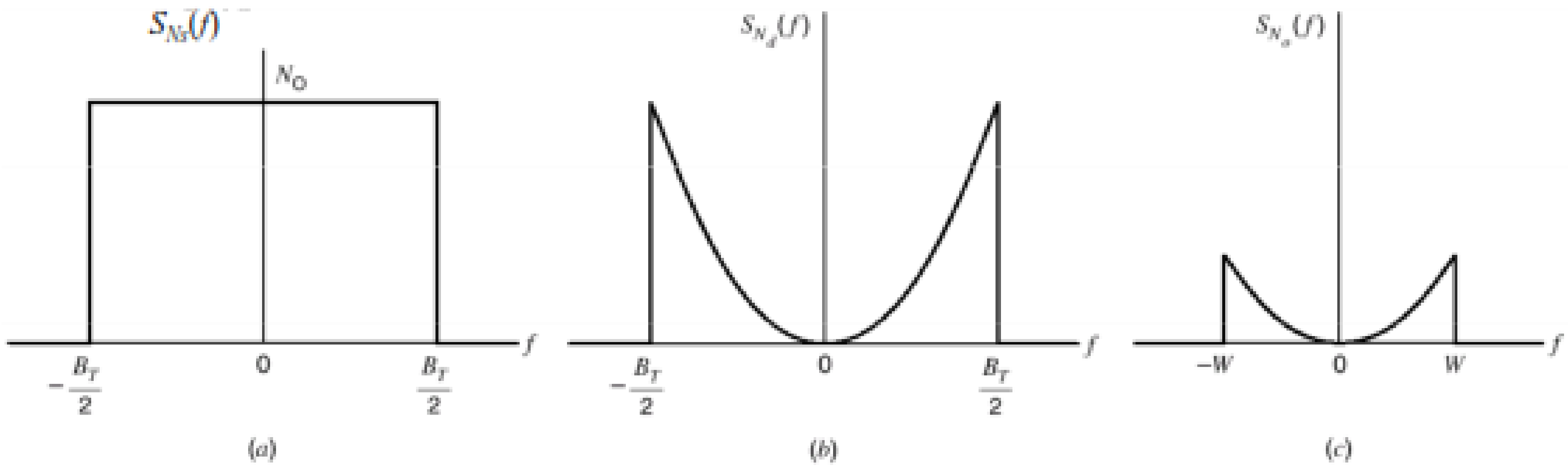
$$\{\text{PSD of } n_d(t)\} = |2\pi f|^2 \times N_0 \qquad |f| \le B_T/2$$

$$\left\{\text{PSD of } f_i(t) = \frac{1}{2\pi A}\frac{dn_s(t)}{dt}\right\} = \left(\frac{1}{2\pi A}\right)^2 |2\pi f|^2 \times N_0 = \frac{f^2}{A^2} N_0$$

- After the LPF, the PSD of noise output $n_o(t)$ is restricted in the band $\pm W$

$$S_{N_o}(f) = \frac{f^2}{A^2} N_0 \qquad |f| \le W$$

Noise power



(a) Power spectral density of quadrature component $n_s(t)$ of narrowband noise $n(t)$.
(b) Power spectral density of noise $n_d(t)$ at the discriminator output.
(c) Power spectral density of noise $n_o(t)$ at the receiver output.

$$S_{N_o}(f) = \frac{f^2}{A^2} N_0 \qquad |f| \le W$$

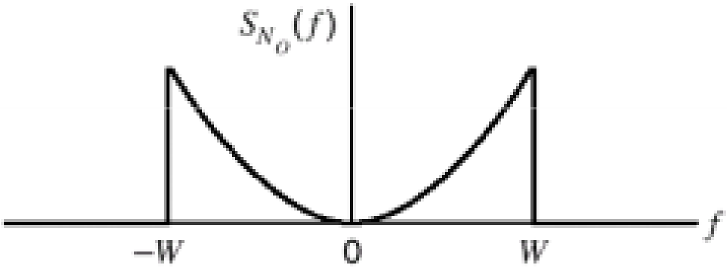
Average noise power at the receiver output:

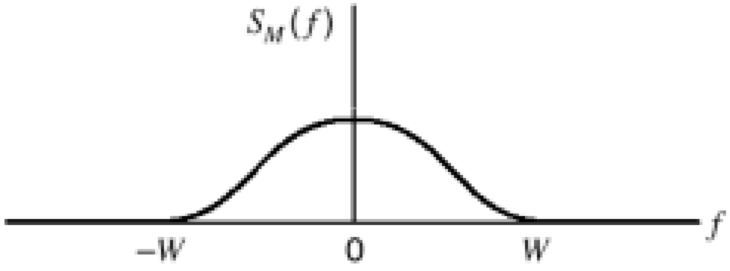
$$P_N = \int_{-W}^{W} \frac{f^2}{A^2} N_0 df = \frac{2N_0 W^3}{3A^2}$$

$$P_N = \int_{-W}^{W} S_{N_o}(f) df$$

Average noise power at the output of FM receiver inversely proportional to A2. As A increases, noise decreases.

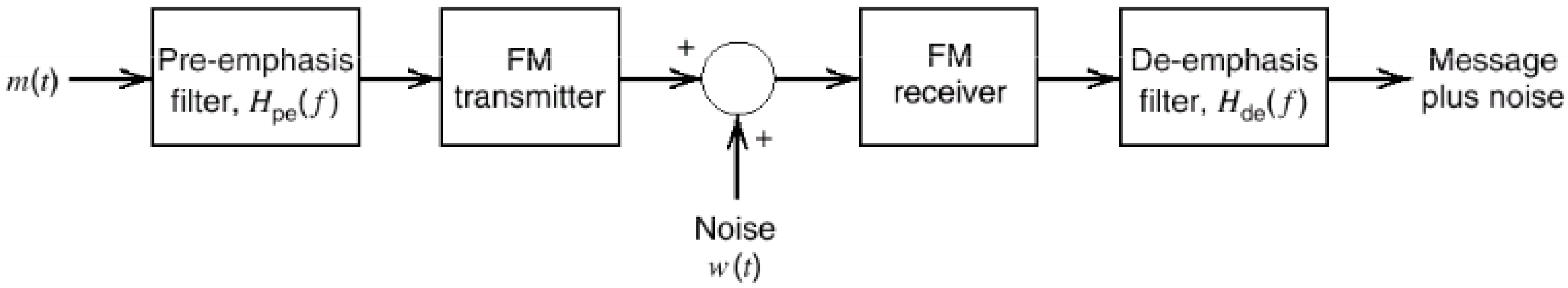
Improve Signal to Noise Ratio





- PSD of the noise at the detector output $\propto$ square of frequency.
- PSD of a typical message typically rolls off at around 6 dB per decade
- To increase SNR_{FM}:
  - Use a LPF to cut-off high frequencies at the output
    - Message is attenuated too, not very satisfactory
  - Use **pre-emphasis** and **de-emphasis**
    - Message is unchanged
    - High frequency components of noise are suppressed

Pre-emphasis and De-emphasis



- $H_{pe}(f)$: used to artificially emphasize the high frequency components of the message prior to modulation, and hence, before noise is introduced.
- $H_{de}(f)$: used to de-emphasize the high frequency components at the receiver, and restore the original PSD of the message signal.
- In theory, $H_{pe}(f) \propto f$, $H_{de}(f) \propto 1/f$.
- This can improve the output SNR by around 13 dB.

Improvement Factor

- Assume an ideal pair of pre/de-emphasis filters

$$H_{de}(f) = 1/H_{pe}(f), \quad |f| \le W$$

- PSF of noise at the output of de-emphasis filter

$$\frac{f^2}{A^2} N_0 \, |H_{de}(f)|^2, \quad |f| \le B_T/2, \quad \left(\text{recall } S_{N_o}(f) = \frac{f^2}{A^2} N_0\right)$$

- Average power of noise with de-emphasis

$$P_N = \int_{-W}^{W} \frac{f^2}{A^2} |H_{de}(f)|^2 N_0 df$$

- Improvement factor (using (7.1))

$$I = \frac{P_N \text{ without pre/de-emphasis}}{P_N \text{ with pre/de-emphasis}} = \frac{\frac{2N_0W^3}{3A^2}}{\int_{-W}^{W} \frac{f^2}{A^2} |H_{de}(f)|^2 N_0 df} = \frac{2W^3}{3\int_{-W}^{W} f^2 |H_{de}(f)|^2 df}$$

Pre-emphasis and De-emphasis Circuits

- (a) Pre-emphasis filter

$$H_{pe}(f) \cong 1 + jf / f_0$$

$$f_0 = 1/(2\pi rC), \qquad R \ll r, \quad 2\pi frC \ll 1$$

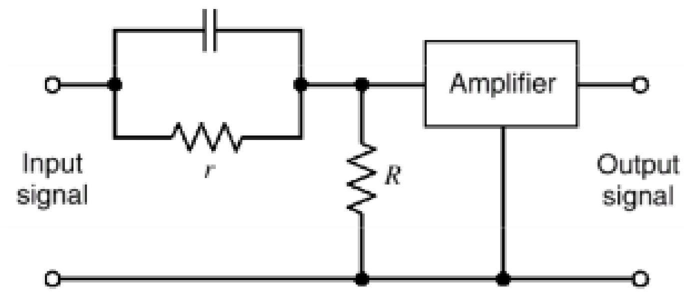
- (b) De-emphasis filter

$$H_{de}(f) = \frac{1}{1 + jf / f_0}$$

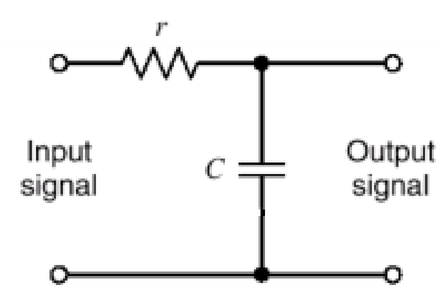
- Improvement

$$I = \frac{2W^3}{3\int_{-W}^{W} f^2 / (1 + f^2 / f_0^2) df} = \frac{(W / f_0)^3}{3[(W / f_0) - \tan^{-1}(W / f_0)]}$$

- In commercial FM, $W = 15$ kHz, $f_0 = 2.1$ kHz
$\Rightarrow I = 22 \Rightarrow 13$ dB (a significant gain)



(a)



(b)

The pre-emphasis filter

- In this experiment, you will be required to investigate the effect of the pre-emphasis filter on the FM demodulated signal.
- You will apply a message signal of constant amplitude but variable frequency to the pre-emphasis filter prior to FM modulation and observe the change in the amplitude of the demodulated signal versus the message signal (you will take 4 different message frequencies).

Phase Modulation

- Angle modulated signal: $s(t) = A_c \cos\big(\omega_c t + \theta(t)\big)$
- For **phase modulation**, the phase is directly proportional to the modulating signal

$$\theta(t) = k_p m(t)$$

- k_p is the phase sensitivity measured in rad/volt. **This quantity is to be measured in the lab**. To measure the phase difference, we apply the unmodulated carrier as one input to the phase comparator and the modulated signal as the second input. We will allow m(t) to vary (assume a dc input) and the phase difference will then vary. We can construct a curve, the slope of which is the constant kp.
- The peak phase deviation is

$$\Delta\theta \quad = k_p \times \max\big(m(t)\big)$$

Phase Modulation

- Phase modulated signal: $s(t) = A_c \cos(\omega_c t + k_p m(t))$
- When $m(t) = A_m \cos \omega_m t$

$$s(t) = A_c \cos(\omega_c t + k_p A_m \cos \omega_m t)$$

- Which looks like an FM signal with $\beta = k_p A_m$
- The spectrum of PM is similar to that for FM.
- The PM can be demodulated using the PLL. However, the **output is proportional to the derivative of m(t).**
- **You can corroborate this result by applying a triangular and rectangular functions to the PM and observing the shape of the demodulated signal.**
- To overcome this problem, the message is integrated prior to being applied to the PM modulator.